**Content Knowledge**

|  | **Foundational** | **Proficient** | **Exemplary** |
| --- | --- | --- | --- |
| **Algebraic Structures** | Define vector space, subspace  Define sums and direct sums  Define linear dependence/independence  Define groups and fields  Understand commutativity, associativity, distributivity  Define spans and bases  Define dimension  Define product space  Define quotient space  Define norm and inner product  Define orthogonal complement | Understand and apply concept of subspace  Connect the concept of direct sum with linear independence  Understand and apply concepts of span and basis  Understand and apply concept of linear combination  Apply concepts of groups and fields  Apply commutativity, associativity, distributivity  Understand and apply concept of dimension  Relate bases, spanning sets, and linearly independent sets  Apply concept of product and quotient spaces  Define affine subset  Define equivalence relation  Give examples of norms  Find orthogonal complements | Prove statements about vector spaces and subspaces  Prove statements about linear independence  Prove statements about spans and bases  Prove statements about sums and direct sums  Prove facts about dimension  Prove statements about eigenspaces  Prove statements about product spaces  Prove statements about quotient spaces  Prove statements about affine subsets |
|
|
|
|
|
|
|
|
| **Linear Transformations** | Define linearity  Understand matrices as linear transformations  Define range and kernel (column space and null space)  Define isomorphism  Define injectivity and surjectivity  Define invariant subspace | Give a geometric interpretation of the determinant  Apply concept of linearity  Understand connection between column space/null space and linear independence  Apply concept of isomorphism  Apply concepts of injectivity and surjectivity  State and apply Rank-Nullity theorem  Find column space and null space for specific matrices  Apply concept of invariance | Prove facts about linear transformations  Prove statements about null space and range  Prove statements about injectivity and surjectivity  Prove statements about isomorphisms  Prove statements about invariant subspaces |
|
|
|
|
|
|
|
| **Vectors and Matrices** | Compute cross product  Compute dot product  Add and multiply matrices  Find the determinant of matrices  Define elementary matrices  Define nonsingular matrices  Use matrices to solve systems of linear equations  Define eigenvectors and eigenvalues  Define diagonalizable matrix  Define similar matrices  Define algebraic and geometric multiplicity  Define conjugate transpose  Define orthonormal vectors  Define self-adjoint matrix  Define normal matrix  Define unitary matrix  *Define singular values* | Give geometric interpretation of cross product  Give geometric interpretation of dot product  Use cross product and dot product to find equations of lines and planes  Understand matrices as linear transformations  Find an inverse matrix from elementary matrices  Find rotation matrices  Use reduced row echelon form to solve systems  Find the eigenvalues of a matrix  Find the eigenvectors of a matrix  Apply algebraic and geometric multiplicity  Find orthonormal bases  Approximate solutions to inconsistent systems using least squares  State Schur’s Theorem  State Spectral Theorem  *State the Singular Value Decomposition*  *Find Singular Value Decompositions* | Deduce properties of the determinant  Use Law of Cosines to interpret dot product  Prove statements about the determinant  Use matrices to represent complex numbers  Derive inverse of 2x2 matrix  Represent linear transformations as matrices  Deduce properties of eigenvalues  Prove statements about eigenvalues and eigenvectors  Prove statements about diagonal matrices  Prove statements about similar matrices  Prove statements about normal and Hermitian matrices  Connect various attributes of matrices  *Approximate solutions to inconsistent systems using the Moore-Penrose pseudoinverse*  *Prove existence of SVD* |
|
|
|
|
|
|
|
|
| **Polynomials** | State the Fundamental Theorem of Algebra  Define characteristic polynomial | Apply the Fundamental Theorem of Algebra | Prove facts about zeroes of polynomials |
| **Proof Skills** | Write valid arguments  Provide counterexamples  Consider converses | Write proofs by contrapositive  Write equality proofs  Write if and only if proofs | Use precise mathematical notation in proofs  Doesn’t prove false statements |

**Skills and Practices**

|  | **Foundational** | **Proficient** | **Exemplary** |
| --- | --- | --- | --- |
| **Explore and Organize** | Student makes use of the data that is presented with the problem, can solve familiar problems; generates some data about the problem/relationship with assistance | Student approaches a new problem and/or explores relationships/ by generating some data about the problem/relationship independently, organizes these data to find order | Student effectively generates data about mathematical situations and organizes these data to find order; identifies promising leads and builds on them; generates original lines of inquiry |
| **Generalize and Test** | Student identifies simple patterns and can describe them informally by finding discrete solutions or relying on intuition | Student identifies patterns and can describe them formally with some justification and testing | Student identifies patterns and processes; describes, tests, and justifies them fully; student looks for patterns between different concepts |
| **Abstract and Symbolize** | Student beginning to decode and use formal notations and definitions | Student can consistently interpret and use many formal notations; defines terms precisely to express mathematical ideas | Student employs a wide variety of formal notations across different mathematical spaces; defines or invents terms to express mathematical ideas |
| **Represent and Connect** | Student represents problems in a single way | Student works shows several different kinds of representation, including computational models when appropriate; can connect and synthesize topics with prompting | Student represents problems in multiple ways; sees math topics as a complete and connected network; computational models are used to gain insights |
| **Retrieve and Strategize** | Student problem solving limited to familiar and current problems; problem solving process is algorithmic and rote rather than strategic | Student remembers and uses specific problem solving techniques/shortcuts; can solve problems in multiple ways | Student retrieves and strategically, creatively, and explicitly employs a variety of problem solving techniques; compares efficacy, efficiency, and elegance of different approaches |
| **Proof** | Student is starting to construct simple mathematical arguments; can explain thinking informally; can provide the building blocks of arguments, either in reasoning/assumptions or in mechanics | Student independently constructs simple mathematical arguments with some formal structure; arguments are mostly complete; can analyze the approaches of others | Student constructs complete mathematical arguments supporting significant conjectures; determines validity of and further develops the arguments of others |
| **Application** | Student work follows problems through to their mathematical conclusion; attempts to apply those results to real world situations and/or new mathematical spaces | Student successfully applies the results of mathematical thinking to address real world situations | Student applies results of mathematical thinking to real world situations at a deeper level; looks for applications independently |
| **Clarity and Articulation** | Student states and shows some work; can communicate part of the process when prompted | Student's work is clear enough to follow manipulations; higher level thinking process more difficult to discern; sometimes takes audience into account | Student's work is succinct and elegant; clearly communicates process and reasoning through multiple modes; effectively targets communication to appropriate audience |
| **Estimation, Precision and Accuracy** | Student addresses accuracy errors with prompting; is beginning to use estimation as a tool to check for reasonability of solutions | Student estimates and checks solutions for reasonability independently; work is mostly correct and precise | Student is precise in all components of their mathematical work; critical thinking about estimation and assumptions is a natural part of their approach |

**Habits of Learning**

|  | **Foundational** | **Proficient** | **Exemplary** |
| --- | --- | --- | --- |
| **Growth Mindset** | Student can meet minimum requirements with assistance and implement familiar procedures; engages with direction; can overcome setbacks with support; starting to ask questions and find personal engagement with material | Student meets minimum requirements and engages independently and consistently; when prompted, explores opportunities to take work to the next level; shows some resilience by seeking general help and using resources appropriately; responds to feedback to improve | Student takes ownership of learning and values self-exploration, intellectual growth, and challenge; approaches work with enthusiasm and takes intellectual risks; views struggle as opportunity for growth; is proactive in using resources flexibly and thoughtfully; seeks out feedback to improve |
| **Community** | Student can collaborate well with some peers; follows the norms, expectations, and rules of the class; participates when called upon | Student looks for opportunities to participate and collaborate effectively in a variety of group settings; student puts in effort to improve classroom culture | Student collaborates effectively with many different peers, soliciting, connecting, and building on the ideas of others; regularly promotes positive classroom culture and attitude about math |
| **Reflection** | Student reflection responds to some part of the prompt and generates some insight about self and/or math | Student reflection engages with the prompt; uses reflection to plan and reach goals | Student reflection yields insights, connections, and specific areas of need; student reflects deeply as part of process beyond specific prompt |
| **Academic Habits** | Student takes notes with support; can find prior work and review it with prompting; completes most assignments; begins class when prompted | Student takes notes independently; finds and reviews past work; completes most assignments on time; ready for class independently and on time | Student organizes prior work and accesses it independently to maintain and deepen understanding; work is turned in on time consistently; is productive and focused on learning for entirety of class |